assumptions made, the experimental error, and the values of A_{eff}^* calculated, the agreement between experimental and calculated thrust could hardly be fortuitous.

Direct deduction of the effect of J upon F and $\dot{m}$ in swirling flow is not possible because of the relationship between J and the swirl velocity: for one injection velocity, each value of J corresponds to a different swirl condition. To separate the effect of J, new swirl velocities were calculated for each A_p based on the assumption that potential vortex flow exists in the entrance to the simulated grain port for which

$$Vr = \text{Const} \tag{3}$$

From the swirl velocities calculated from Eq. (3) for each A_p and each injection velocity, reference spin rates were calculated. Results are given in Fig. 6, which shows that, at one spin rate, J may have a substantial effect on thrust (and $\dot{m}$). The effect is complicated: its direction and magnitude depend upon spin rate, value of J, and type of nozzle system. The reference spin rates calculated here should not be applied to systems of different scale without attention to appropriate similarity relations.

Conclusions

Tangential injection provided a method of studying swirling effects. The data obtained also allowed study of axial flow in multiple nozzles displaced various distances from the centerline. The data permit several significant conclusions: 1) As swirl intensity increases, both mass flow rate ($\dot{m}$) and thrust (F) are substantially reduced. These reductions of F and $\dot{m}$ can be explained quantitatively, using one-dimensional isentropic flow theory, as being due to an effective reduction in throat area caused by development of a low-density core. 2) For a given reference spin rate, as $J = A^*/A_p$ decreases (tangential velocity at the port radius increases), the effects depend on the type of nozzle system and the values of J and spin rate. 3) For flow in which swirl is essentially absent, thrust is slightly reduced as nozzles are displaced farther from the swirl chamber centerline for any J. The total thrust reduction is primarily a result of a total pressure drop.

The results depict the behavior of swirling and axial flow through multiple nozzles; however, methods are needed which will permit analytical determination of vortex core size and effective throat area in multiple nozzles as the analyses of Mager[2] and Norton et al.[5] have made possible for single nozzles.

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Deep Space Shuttles: Chemical vs Nuclear; Direct Flight vs Near-Orbit Rendezvous

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Nomenclature

C = standard transportation cost/lb of payload in orbit
g = acceleration of gravity
I = specific impulse
r = vehicle gross weight/burnout weight, $e^{\Delta v/gI}$
w = weight
v = ideal velocity gain
α = (cost/lb to place a stage in near orbit when it aids in its own delivery)/C_b
β = (re-entry vehicle plus payload weight)/(payload weight)
λ = stage structure factor, $w_{str}/(w_{pr} + w_{str})$
ν = (stage manufactured hardware cost/lb)/(effective boost cost/lb in orbit: C_b, αC_b, or $\alpha\beta C_b$)

Subscripts

E = expendable
R = reusable
a = ascent from near orbit to deep space orbit
b = boost to near orbit
e = direct descent to Earth
g = stage gross
n = descent to near orbit refueling
p = stage payload
pr = stage propellant
str = stage structure

Introduction

As space activities mature, and space transportation costs are reduced, it appears likely that there will be growing interest in space systems operating at ever greater distances from the Earth, in geosynchronous orbits and beyond. It is therefore appropriate to consider advanced transportation modes for delivery, return, maintenance and logistic support of deep space systems. Currently, only two alternative modes are receiving serious attention: 1) non re-enterable chemical shuttles which depart from near orbit, transfer to deep space orbit, and then return again to near orbit; and 2) solid core nuclear shuttles which operate in the same manner.

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Table 1 Operating modes and vehicle types

Ascent/descent modes	One-way payloads: No descent	One-way payloads: Descent to near-orbit rendezvous	Round-trip payloads: Direct descent to Earth	Round-trip payloads: Descent to near-orbit rendezvous
Direct ascent from Earth	E(chem)[a]	...	E(chem) R(chem[b]; nucl)	...
Ascent from near orbit	E(chem)	R(chem; nucl)	E(chem) R(chem; nucl)	R(chem; nucl)

[a] E = expendable; R = reusable.
[b] Performance inadequate.

The purpose of this analysis is to obtain a preliminary comparison between these modes as well as several other approaches which also appear to merit consideration. The major alternatives are as follows: 1) payload type (one-way; or round-trip); 2) vehicle type—a) propulsion (chemical; or nuclear), b) reusability (full; or partial; or expendable); 3) flight mode—a) ascent (direct from Earth-launched booster; or from near-orbit refueling), b) descent (none; or direct to earth; or to rendezvous in near orbit).

The alternatives of primary interest are summarized in Table 1, which is the result of several simplifying assumptions. First, direct descent for reusable shuttles after delivering one-way payloads has been dropped, since the analysis is complex and the relationships between alternatives are quite similar to the round-trip payload case. Second, partial reusability has been dropped since the results depend heavily on engineering and cost studies which are beyond the scope of this analysis. Third, it is assumed that nuclear approaches must in all cases be reusable if they are to be cost effective. The possibilities of multistage approaches and refueling in deep space are considered beyond the first-generation modes postulated, in terms of system and operational complexity.[1]

It is believed that after these simplifying assumptions, the remaining array of eleven alternatives shown in Table 1, all of which are considered below, provide a sufficient basis for useful comparisons.

Analysis

Ascent from near-orbit refueling

In this mode, it is assumed that there is an operational station in near orbit where turnaround, refueling and checkout functions are performed. Propellants are delivered from the earth to this station by an Earth-based shuttle at a cost/lb of C_b.

First, consider an expendable stage which is launched from near orbit to place a payload w_p in deep space orbit. This vehicle can be described by the relation $(w_g + w_p)/(\lambda w_g + w_p) = r_a$, which can be solved to give

$$w_g/w_p = (r_a - 1)/(1 - \lambda r_a) \tag{1}$$

Second, consider a reusable stage which is launched from near orbit to place a payload w_p in deep space orbit, and then return without payload to near orbit. The vehicle can be described by the following equations

$$(w_g + w_p)/(\lambda w_g + w_{prn} + w_p) = r_a \tag{2}$$

$$(\lambda w_g + w_{prn})/\lambda w_g = r_n \tag{3}$$

$$\lambda w_g + w_{pra} + w_{prn} = w_g \tag{4}$$

Since by definition $r_a = r_n$, Eqs. (2–4) can be solved to give

$$w_g/w_p = (r_a - 1)/(1 - \lambda r_a^2) \tag{5}$$

Third, consider a reusable vehicle such as the aforementioned, but which carries a round-trip payload w_p. This vehicle can be described by Eqs. (2) and (4), with Eq. (3) replaced by

$$(\lambda w_g + w_{prn} + w_p)/(\lambda w_g + w_p) = r_n \tag{6}$$

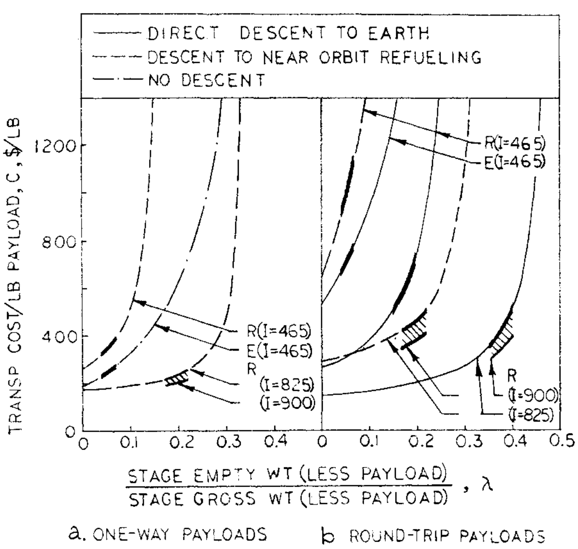

Fig. 1 Ascent from near-orbit comparison of delivery cost vs structure factor for several deep space shuttle modes.

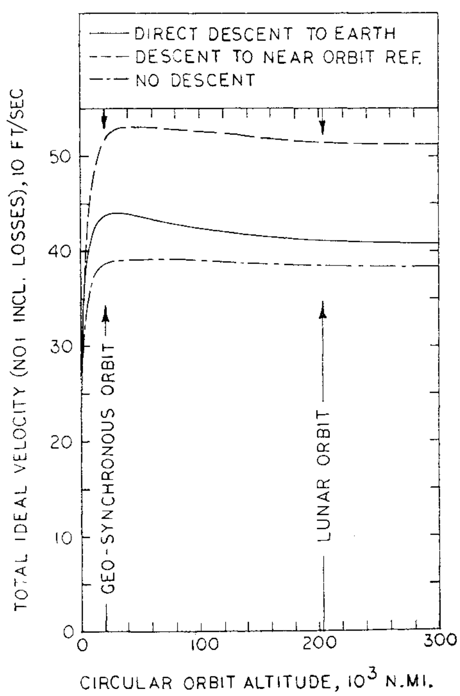

Fig. 2 Velocity requirements for deep space flights between the Earth and circular orbits.

Equations (2, 4, and 6) solve to give

$$w_g/w_p = (r_a^2 - 1)/(1 - \lambda r_a^2) \quad (7)$$

Using Eqs. (1, 5, and 7), the following expressions can be written for the cost of delivering a unit payload ($w_p = 1$) by the several modes in question:
one-way payloads

$$C_{E,a} = \alpha C_b\{[(r_a - 1)/(1 - \lambda r_a)](1 + \nu_a\lambda) + 1\} \quad (8)$$

$$C_{R,n} = C_b\{[(r_a - 1)/(1 - \lambda r_a^2)](1 - \lambda) + 1\} \quad (9)$$

round-trip payloads

$$C_{R,e} = \alpha C_b[(r_{a,e} - 1)/(1 - \lambda r_{a,e}) + 1] \quad (10)$$

$$C_{E,e} = \alpha\beta C_b\{[(r_{a,e} - 1)/(1 - \lambda r_{a,e})](1 + \nu_{a,e}\lambda) + 1\} \quad (11)$$

$$C_{R,n} = C_b\{[(r_a^2 - 1)/(1 - \lambda r_a^2)](1 - \lambda) + 1\} \quad (12)$$

Equations (8–12) assume that for all reusable vehicles, the number of reuses is sufficiently high (say, 100) and the refurbishment and turnaround costs sufficiently low (say $\sim$1% of unit cost), that amortized hardware plus turnaround cost per flight is negligible compared to the cost of one flight's worth of propellant delivered to near orbit. This assumption should be valid unless $C_b \ll \$100$/lb.

Transportation costs per pound of payload for the various modes as computed from Eqs. (8–12) are shown in Fig. 1 as a function of λ, with the most probable ranges of λ indicated by the heavy segments of the curves. The following numerical values were assumed for the other variables: $C_b =$ \$100/lb; I(chem) = 465 sec; I(nucl) = 825 and 900 sec; $\Delta v_a = \Delta v_n =$ 14,000 fps (adequate for all geocentric and lunar orbits); $\Delta v_e =$ 5000 fps (also adequate for all geocentric and lunar orbits); $\alpha = 0.8$; $\beta = 2.0$; $\nu_a\alpha C_b = \nu_{a,e}\alpha\beta C_b =$ expendable manufactured hardware cost = \$300/lb.

The values of 14,000 fps for $\Delta v_a = \Delta v_n$, and 5000 fps for Δv_e, on which the results of Fig. 1 are based, are more than adequate for circular orbits up to nearly geosynchronous altitude, and beyond altitudes of about 50,000 naut miles, as shown in Fig. 2. This is particularly true for the direct descent mode, the velocity requirements for which fall off with altitude more rapidly than for the other modes. The maximum values of Δv_a, Δv_n and Δv_e have been used because of the special interest in geosynchronous orbits, which happen to be near the requirement maxima in all cases.

The values assumed for C_b, I, β and $\nu_a\alpha C_b = \nu_{a,e}\alpha\beta C_b$ are thought to be consistent with current technology, and not subject to sufficiently wide variations of estimates, as to affect the results significantly.

The parameter α, which appears in all direct descent cases, reflects the fact that on each flight the deep space shuttle vehicle can help place itself into near orbit, by acting in place of the orbiter element of a two-stage Earth-to-near-orbit shuttle. Thus, the orbiter contribution (or its equivalent) to C_b can be eliminated in these cases, reducing C_b by the amount $(1 - \alpha)$. The selection of 0.8 as a value for α is thought to introduce a conservative bias in regard to the relative costs of direct descent, since the orbiter probably contributes more than 0.2 C_b to the launch cost.

Direct ascent from Earth-launched booster

It is conceivable that evolving diplomatic or military circumstances could render operations in near orbit, just described, undesirable or infeasible for certain missions. In such an eventuality, it might be necessary to perform deep space shuttle flights by direct ascent from an Earth-launched booster, with direct return to Earth.

The three modes of accomplishing this which are shown in Table 1, are not amenable to the above kind of analytic treatment, since the mission performance requirement depends heavily on the Δv contributed by the booster, which may vary considerably from case to case. Some indication of the feasibility and cost effectiveness of these modes can be obtained, however, by assuming across the board a booster of standard size and performance, say, w_g = 2.7 million lb, λw_g = 0.4 million lb, I(ave) = 420 sec, and total cost/flight = \$3 million. This corresponds roughly to the first stage of a two-stage fully reusable Earth-based shuttle such as currently being studied.

Preliminary analysis indicates that if this booster were used as a launch platform for an expendable chemical ascent vehicle ($\lambda = 0.07$; I = 465 sec) weighing 400,000 lb (approx. optimum), then this vehicle combination can place about 40,000 lb in the most stringent geocentric polar orbit, at a cost of about \$280/lb, based on the numerical assumptions of the preceding analysis. If return to Earth is desired, then this 40,000 lb would consist of a reenterable spacecraft containing integral de-orbit propulsion for 3000–5000 fps, and carrying a useful payload of about 12,000 lb, for a total round-trip cost of about \$900/lb. If the standard booster were used as a launch platform for a reenterable, reusable nuclear shuttle vehicle ($\lambda = 0.35$; I = 825 sec) weighing 400,000 lb (approx. optimum), then this vehicle combination could carry about 12,000 lb through the round-trip flight at a total round-trip cost of about \$400/lb.

These cost results are computed from point-design analyses of the vehicles involved [not from Eqs. (8–12)], and include the \$3 million cost/flight assumed for the standard booster.

In each of these three cases, the cost/lb is about the same or somewhat greater, than the cost for the corresponding mode using near orbit refueling.

Observations

1) For delivering one-way payloads such as propellants for refueling, either from near orbit or direct from an Earth-launched booster, expendable chemical propulsion appears to be the preferable mode, all factors considered. The marginal economic advantages offered by reusable nuclear shuttles in this application, would probably be at least counterbalanced by added operational complexity and risks resulting from the radiation environment around the nuclear engines.

2) For round-trip payloads delivered from near orbit, nuclear approaches appear to be clearly economically preferable to expendable chemical modes. Fully reusable chemical vehicles capable of direct return to earth, however, appear competitive if λ can be held to $\sim$0.15 or less. (Here, partially reusable chemical approaches may be promising and should be investigated.) The most economical mode appears to be a reusable nuclear vehicle designed for direct return to Earth.

3) For round-trip payloads delivered by direct ascent from an earth-launched booster, a partially reusable chemical vehicle appears adequate to perform the mission. Here also, however, the most economical approach seems to be a reusable Earth-based nuclear upper stage.

4) While direct flight appears to impose somewhat higher costs than rendezvous in near orbit, its benefits in terms of operational flexibility and simplicity, could justify it even in the absence of absolute political or military constraints.

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